

Sections 4.5 – I.C.E. – Translations of Sine & Cosine

Translations of Sine and Cosine

If we have the constant c in the general equations,

$$y = A \sin B(x - C) + D$$

and

$$y = A \cos B(x - C) + D$$

then these equations have the following characteristics: **amplitude** = $|a|$

$$\text{period} = \frac{2\pi}{B}$$

1. If $C > 0$ there is a horizontal shift C units to the right and if $C < 0$ there is a horizontal shift C units to the left.
2. If $D > 0$ the shift is d units upward and if $D < 0$ the shift is d units downward.
3. If $A < 0 \rightarrow$ reflection across x -axis.
4. If $B < 0 \rightarrow$ reflection across y -axis.

1) Find the amplitude, frequency, and period of the following equations:

a) $y = -4 \sin \frac{x}{6}$

amplitude = 4

frequency = $\frac{1}{6}$

period = $\frac{2\pi}{1/6} = 12\pi$

b) $y = 9 \cos 5 \left(x + \frac{7\pi}{8} \right)$

amplitude = 9

frequency = 5

period = $\frac{2\pi}{5}$

c) $y = 2 \sin 4x - 5$

amplitude = 2

frequency = 4

period = $\frac{2\pi}{4} = \pi/2$

d) $y = -\frac{1}{3} \cos \frac{1}{3} x - \frac{1}{3}$

amplitude = $\frac{1}{3}$

frequency = $\frac{1}{3}$

period = $\frac{2\pi}{1/3} = 6\pi$

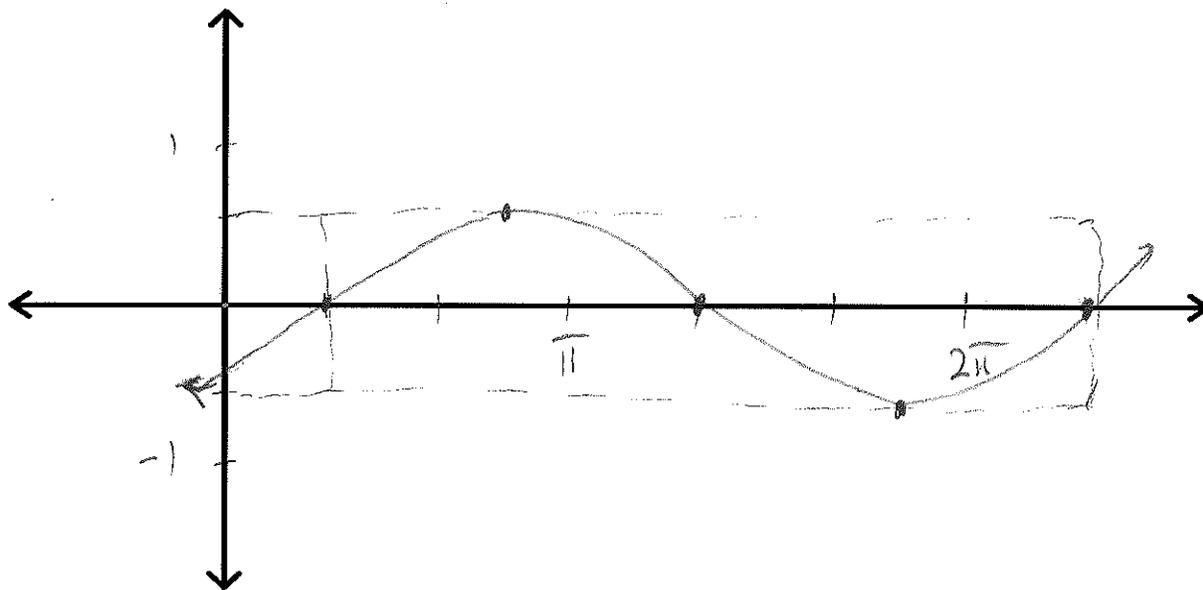
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Sketch graphs of the following: (Do either 1 full cycle or 0 to 2π)

2) $y = \frac{1}{2} \sin\left(x - \frac{\pi}{3}\right)$

$a = \frac{1}{2}$

horizontal shift $\frac{\pi}{3}$ units right



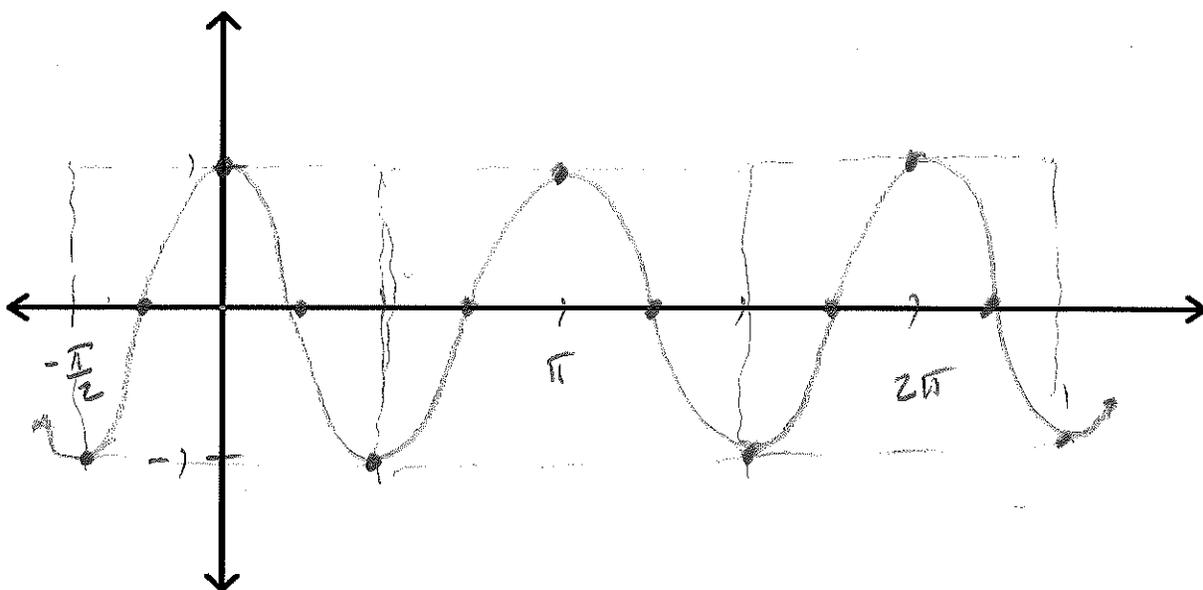
3) $y = -\cos 2\left(x + \frac{\pi}{2}\right)$

$a = 1$

$P = \frac{2\pi}{2} = \pi$

horizontal shift left $\frac{\pi}{2}$

reflects over x-axis

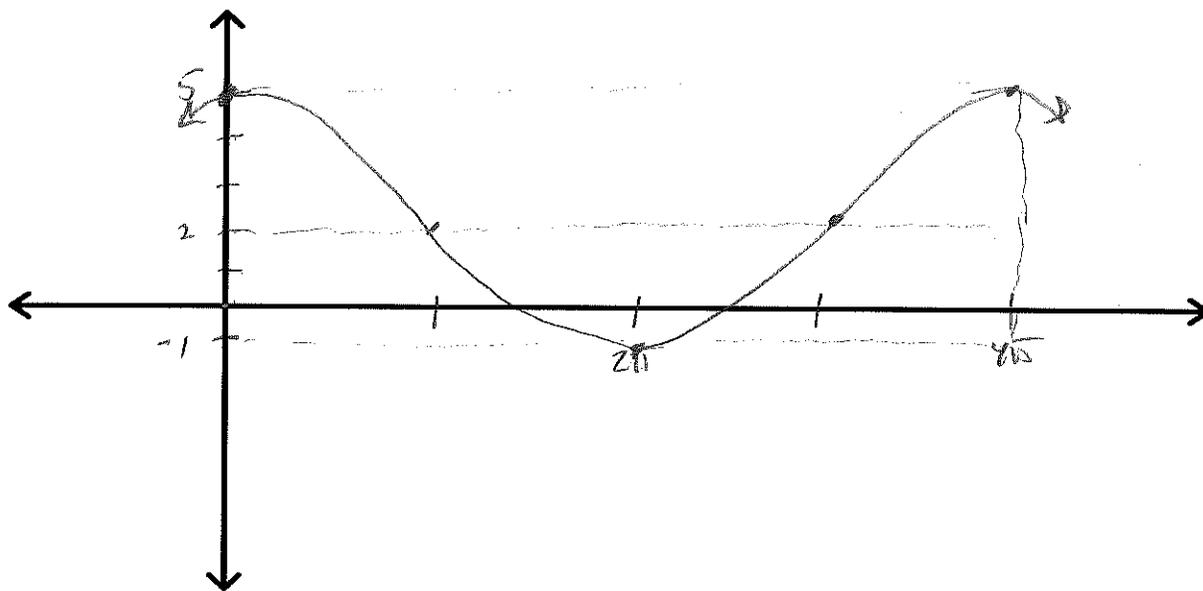


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4) $y = 3\cos\frac{1}{2}x + 2$

amp = 3
 $P = 2\pi / \frac{1}{2} = 4\pi$

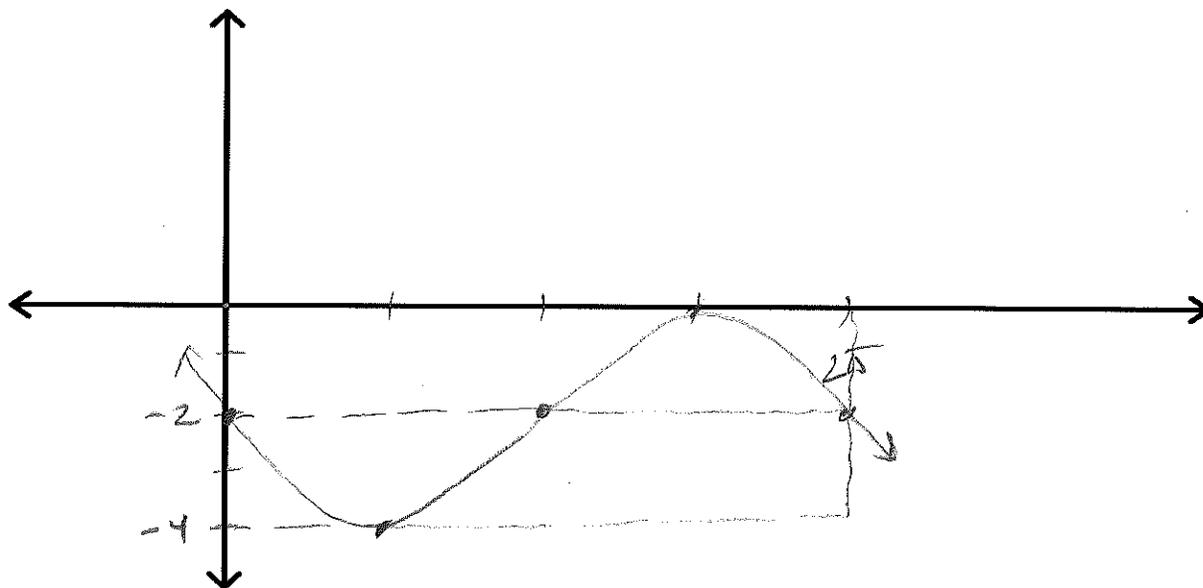
vertical shift up 2



5) $y = -2\sin x - 2$

amp = 2
 $P = 2\pi$

vertical shift down 2
 reflect over $y = -2$

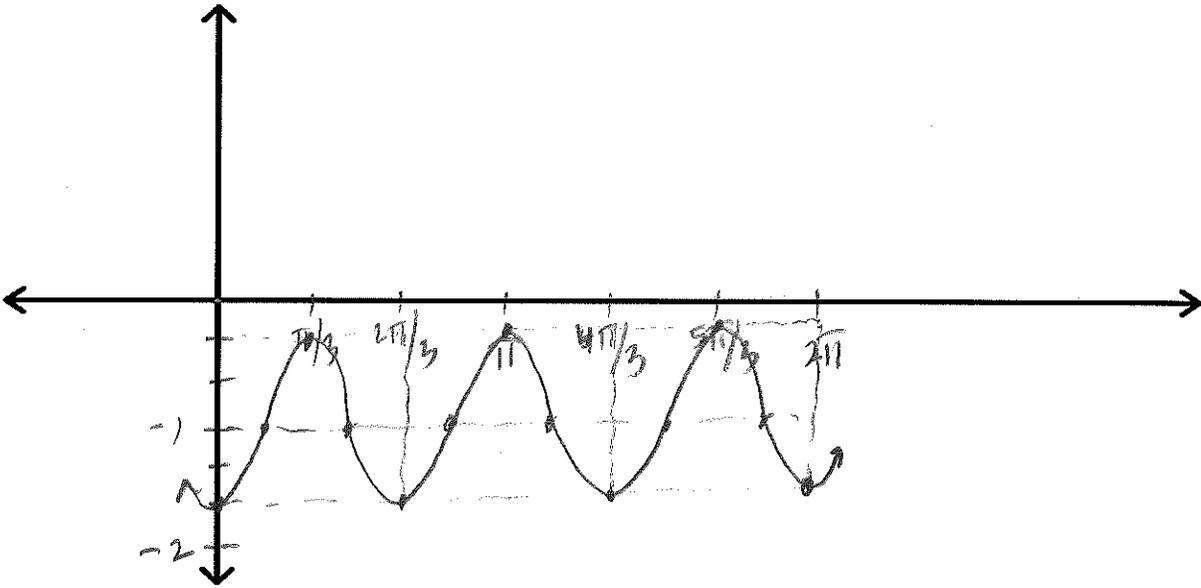


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6) $y = -\frac{2}{3}\cos 3x - 1$

amp = $\frac{2}{3}$
 $P = \frac{2\pi}{3}$

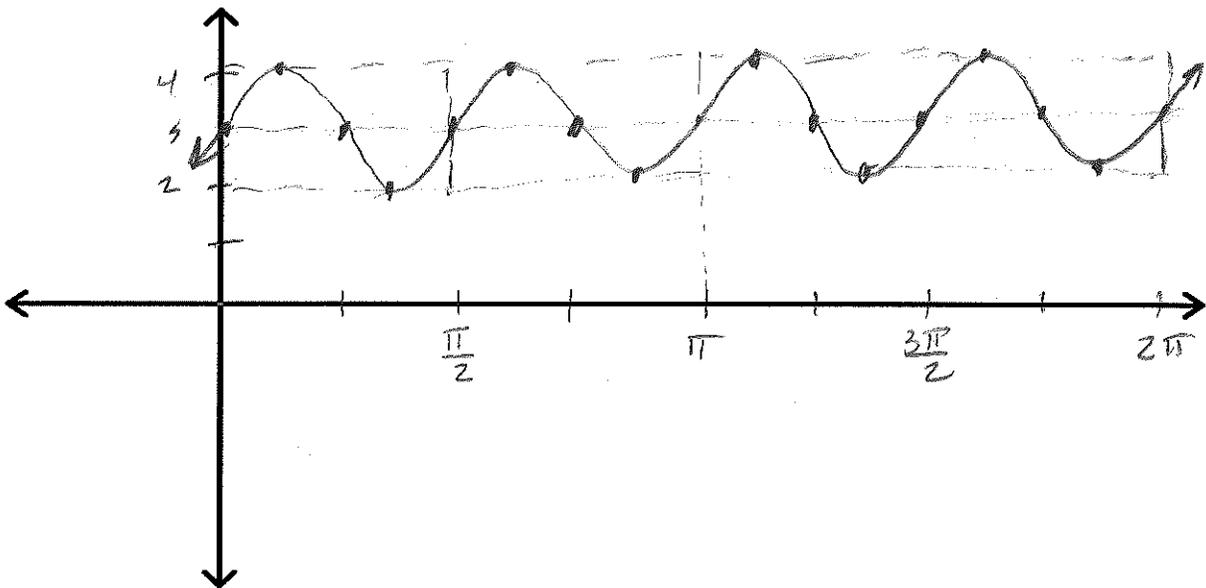
vertical shift down 1
 reflect over $y = -1$



7) $y = \sin 4(x - \pi) + 3$

$P = \frac{2\pi}{4} = \frac{\pi}{2}$
 horizontal shift right π

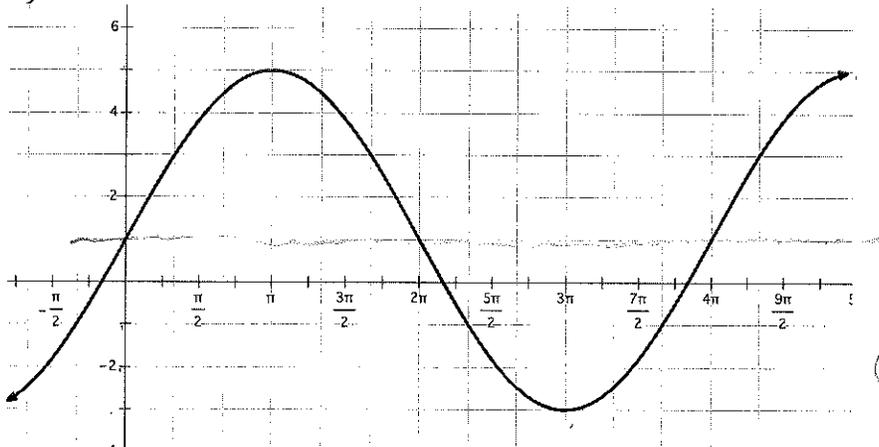
vertical shift up 3



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Identify an equation (sine or cosine) for the following graphs:

8)



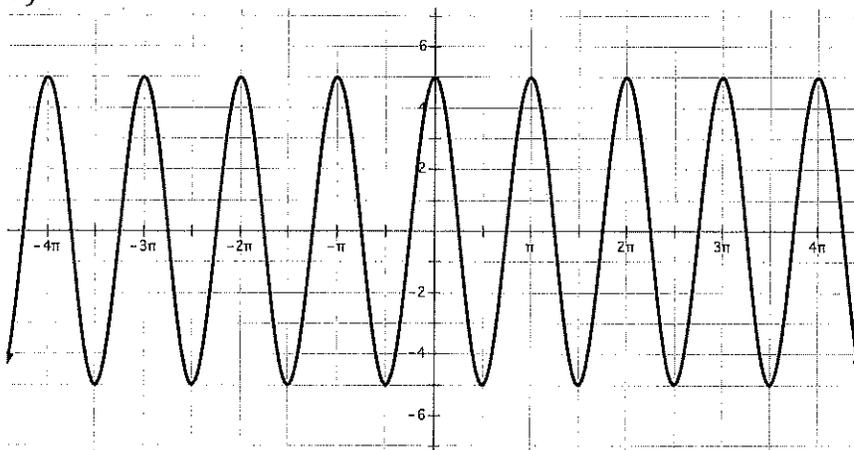
$$\begin{aligned} \text{new center} &= 1 \\ a &= 4 \\ P &= 4\pi = \frac{2\pi}{B} \\ &\Rightarrow B = \frac{1}{2} \end{aligned}$$

$$y = 4 \sin \frac{x}{2} + 1$$

or

$$y = 4 \cos \frac{1}{2}(x - \pi) + 1$$

9)



$$\begin{aligned} a &= 5 \\ P &= \pi = \frac{2\pi}{B} \\ &\Rightarrow B = 2 \end{aligned}$$

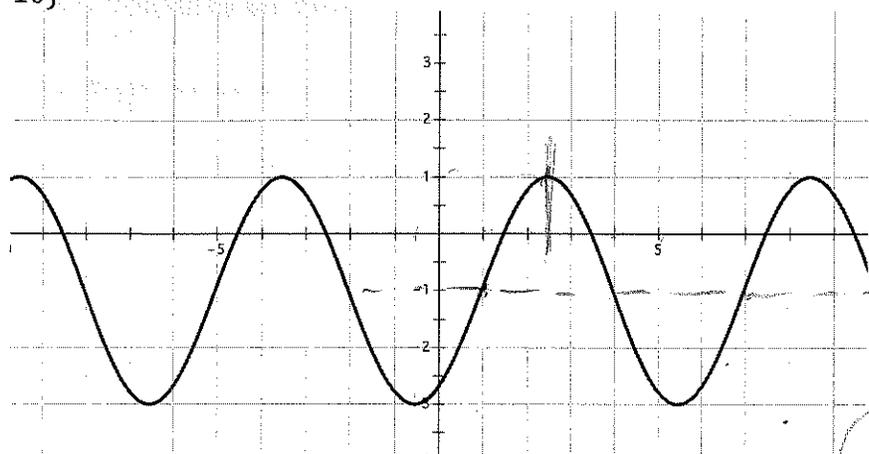
$$y = 5 \cos 2x$$

or

$$y = 5 \sin 2(x + \pi/4)$$

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10)



new center = -1

$$a = 2$$

$$P = b = \frac{2\pi}{B}$$

$$\Rightarrow B = \frac{\pi}{3}$$

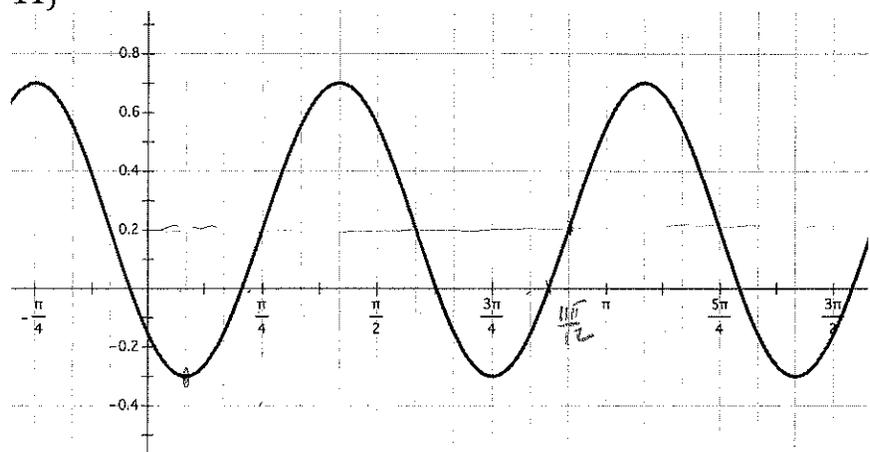
horizontal shift right 1 unit

$$y = 2 \sin \frac{\pi}{3} (x - 1) - 1$$

or

$$y = 2 \cos \frac{\pi}{3} (x + \frac{5}{2}) - 1$$

11)



new center = 1/5

$$a = .5$$

horizontal shift right $\pi/4$

$$y = \frac{1}{2} \sin 3(x - \pi/4) + \frac{1}{5}$$

or

$$y = \frac{1}{2} \cos 3(x + \pi/4) + \frac{1}{5}$$

$$P = \frac{11\pi}{12} - \frac{3\pi}{12} = \frac{8\pi}{12} = \frac{2\pi}{3} = \frac{2\pi}{B}$$

$$\Rightarrow B = 3$$